Übungsblatt Ableitung der ln-Funktion

$f(x) = \ln\left(\frac{3}{x}\right)$ *kann man ableiten nach der Kettenregel:*

$f'(x) = \frac{1}{\frac{3}{x}} \cdot \left(-\frac{3}{x^2}\right)$

$= -\frac{x}{3} \cdot \frac{3}{x^2} = -\frac{1}{x}$

Einfacher geht's mit der Zerlegung:

$f(x) = \ln(3) - \ln(x)$; also

$f'(x) = -\frac{1}{x}$

3 Bilden Sie die 1. Ableitung.

a) $f(x) = 1 + \ln(x)$ b) $f(x) = x + \ln(x)$ c) $f(x) = 2x + \ln(2x)$ d) $f(x) = x^2 + \ln(tx)$

e) $f(x) = \ln\left(\frac{1}{x}\right)$ f) $f(x) = \ln\left(\frac{t}{x}\right)$ g) $f(t) = \ln\left(\frac{t}{x}\right)$ h) $f(t) = \ln(t + x)$

4 a) $f(x) = \ln\left(\sqrt{x}\right)$ b) $f(x) = \ln(1 + 3x^2)$ c) $f(x) = \ln(1 - x^2)$ d) $f(x) = 3\ln\left(\sqrt{4x}\right)$

e) $f(x) = \sqrt{\ln(x)}$ f) $f(x) = \ln(\sin(x))$ g) $f(x) = \sin(\ln(x))$ h) $f(x) = (\ln(x))^{-1}$

5 Bilden Sie die 1. und 2. Ableitung.

a) $f(t) = \ln(t^4)$ b) $f(t) = (\ln(t))^4$ c) $f(u) = \ln\left(\frac{u}{u+1}\right)$ d) $f(t) = k \cdot \ln\left(\sqrt[3]{2t}\right)$

e) $f(s) = (\ln(s - a))^3$ f) $f(x) = x \cdot \ln(x)$ g) $f(x) = \frac{1}{\ln(x)}$ h) $f(x) = \frac{x}{\ln(x)}$

Lösungen

3

a) $f(x) = 1 + \ln(x);\ f'(x) = \frac{1}{x}$

b) $f(x) = x + \ln(x);\ f'(x) = 1 + \frac{1}{x}$

c) $f(x) = 2x + \ln(2x);\ f'(x) = 2 + \frac{1}{x}$

d) $f(x) = x^2 + \ln(tx);\ f'(x) = 2x + \frac{1}{x}$

e) $f(x) = \ln\left(\frac{1}{x}\right);\ f'(x) = -\frac{1}{x}$

f) $f(x) = \ln\left(\frac{t}{x}\right);\ f'(x) = -\frac{1}{x}$

g) $f(t) = \ln\left(\frac{t}{x}\right);\ f'(t) = \frac{1}{t}$

h) $f(t) = \ln(t + x);\ f'(t) = \frac{1}{t + x}$

4

a) $f(x) = \ln\left(\sqrt{x}\right) = \frac{1}{2}\ln(x);\ f'(x) = \frac{1}{2x}$

b) $f(x) = \ln(1 + 3x^2);\ f'(x) = \frac{6x}{1 + 3x^2}$

c) $f(x) = \ln(1 - x^2);\ f'(x) = \frac{-2x}{1 - x^2} = \frac{2x}{x^2 - 1}$

d) $f(x) = 3 \cdot \ln\left(\sqrt{4x}\right) = 3 \cdot \ln\left(2\sqrt{x}\right);\ f'(x) = 3 \cdot \frac{1}{2\sqrt{x}} \cdot \frac{2}{2\sqrt{x}} = \frac{3}{2x}$

e) $f(x) = \sqrt{\ln(x)};\ f'(x) = \frac{1}{2\sqrt{\ln(x)}} \cdot \frac{1}{x}$

f) $f(x) = \ln(\sin(x));\ f'(x) = \frac{\cos(x)}{\sin(x)}$

g) $f(x) = \sin(\ln(x));\ f'(x) = \frac{1}{x} \cdot \cos(\ln(x))$

h) $f(x) = (\ln(x))^{-1};\ f'(x) = -1 \cdot (\ln(x))^{-2} \cdot \frac{1}{x} = -\frac{1}{x \cdot (\ln(x))^2}$

5

a) $f(t) = \ln(t^4) = 4 \cdot \ln(|t|);\ f'(t) = \frac{4}{t};\ f''(t) = -\frac{4}{t^2}$

b) $f(t) = (\ln(t))^4;\ f'(t) = \frac{4}{t} \cdot (\ln(t))^3;\ f''(x) = -\frac{4}{t^2} \cdot (\ln(t))^3 + \frac{12}{t^2} \cdot (\ln(t))^2 = \frac{4}{t^2} \cdot (\ln(t))^2 \cdot (-\ln(t) + 3)$

c) $f(u) = \ln\left(\frac{u}{u+1}\right);\ f'(u) = \frac{1}{\frac{u}{u+1}} \cdot \frac{(u+1) \cdot 1 - u \cdot 1}{(u+1)^2} = \frac{u+1}{u} \cdot \frac{1}{(u+1)^2} = \frac{1}{u(u+1)};\ f''(u) = -\frac{2u+1}{u^2 \cdot (u+1)^2}$

d) $f(t) = k \cdot \ln\left(\sqrt[3]{2t}\right) = k \cdot \frac{1}{3} \cdot \ln(2) + \ln(t));\ f'(t) = k \cdot \frac{1}{3t};\ f''(t) = -\frac{k}{3t^2}$

e) $f(s) = (\ln(s - a))^3;\ f'(s) = 3 \cdot (\ln(s - a))^2 \cdot \frac{1}{s - a};$

$f''(s) = 6 \cdot \ln(s - a) \cdot \frac{1}{s - a} \cdot \frac{1}{s - a} + 3 \cdot (\ln(s - a))^2 \cdot \frac{-1}{(s - a)^2} = \frac{3 \cdot \ln(s - a) \cdot (2 - \ln(s - a))}{(s - a)^2}$

f) $f(x) = x \cdot \ln(x);\ f'(x) = \ln(x) + 1;\ f''(x) = \frac{1}{x}$

g) $f(x) = \frac{1}{\ln(x)};\ f'(x) = \frac{-1}{x \cdot (\ln(x))^2};\ f''(x) = \frac{(\ln(x))^2 + x \cdot 2\ln(x) \cdot \frac{1}{x}}{x^2 \cdot (\ln(x))^4} = \frac{\ln(x) + 2}{x^2 \cdot (\ln(x))^3}$

h) $f(x) = \frac{x}{\ln(x)};\ f'(x) = \frac{\ln(x) - 1}{(\ln x)^2};\ f''(x) = \frac{(\ln(x))^2 \cdot \frac{1}{x} - (\ln(x) - 1) \cdot 2\ln(x) \cdot \frac{1}{x}}{(\ln(x))^4} = \frac{2 - \ln(x)}{x \cdot (\ln x)^3}$